PP-30

UNLOADED Q'S OF AXIALLY ASYMMETRIC MODES OF DIELECTRIC RESONATORS

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ABSTRACT

Resonant frequencies and unloaded Q's of hybrid modes on cylindrical, dielectric resonators in conducting enclosures are studied. The formulation uses the mode matching technique, and includes axially symmetric (TE_{0m} and /or TM_{0m}), as well as axially non-symmetric hybrid (HE_{nm}) modes which could exist in dielectric loaded waveguides. The structures analyzed can simulate resonators on microstrip substrates.

The losses in both the conducting walls and the dielectric material, and stored energy in the general resonator are obtained. Mode amplitudes determined from the solution of the boundary value problem by mode matching, are used to rigorously compute the unloaded Q's of the resonators.

Numerical results in the millimeter wave range are presented for the unloaded Q's of various modes, as a function of the resonator parameters. From these results effect of losses on the resonator wall are discussed, and methods of improving the unloaded Q's are explored. Experimental results are presented which verify the accuracy of the model used and the numerical computations.

1. Introduction

Dielectric resonator applications at microwave and millimeter wave frequencies require accurate characterization of the resonant frequency, unloaded Q's and coupling coefficients of the resonators to other resonators and to transmission media. Considerable work has been done in recent years on the analysis of dielectric resonators [1]-[3]. This paper presents an accurate method for the determination of the resonant frequencies and unloaded Q's of cylindrical dielectric resonators, excited in axially symmetric or asymmetric (hybrid) modes. The configurations analyzed can accurately model the resonators characteristic when used in an MIC environment, taking account of the effects of substrates, enclosures and ground planes. Such characterization is not available in the literature except for the $TE_{01\delta}$ and $TM_{01\delta}$ modes [4] or for hybrid modes with dielectric extending all the way between the end planes [6]. At higher microwave and millimeter wave frequencies it is important to select the appropriate modes that yield optimum unloaded Q's. This selection can be done based on analysis such as that presented in this paper.

Section 2 introduces the geometrical configuration of the model being analyzed, summarizes the analysis procedure using mode matching, and indicates the loss contribution due to each element of the structure (i. e. resonator, substrate, enclosure's conducting walls and ground planes). Numerical results obtained from the analysis, for several representative cases are given in Section 3. Experimental measurements results are also included to verify the accuracy of the analysis.

2. Analysis

The configuration being analyzed is shown in Fig. 1. The dielectric resonator of radius a, length ℓ and relative dielectric constant ϵ_{r_1} is placed on top of a microstrip line substrate of thickness t and relative dielectric constant ϵ_{r_2}. The enclosure has radius b and length L. Since the structure is uniform in azimuth, the fields in each of the three regions I, II and III shown in Fig. 1, will have the same angular dependence $e^{\pm jn\phi}$, where $n = 0, 1, \ldots$ Electromagnetic fields in regions I and III can be expressed as a linear combination of normal modes (TE_{mn} and TM_{mn})

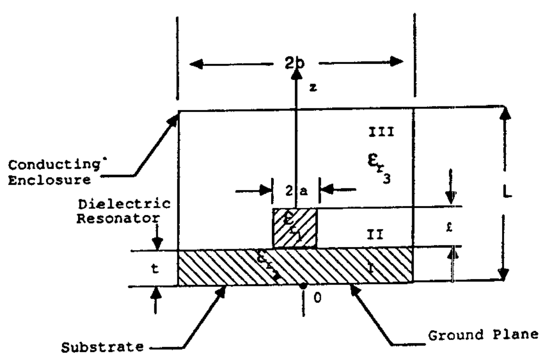

Fig. 1 Dielectric resonator over microstrip substrate.

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of circular waveguides of radius b filled with homogeneous dielectrics of relative dielectric constants ϵ_{r_2} and ϵ_{r_3} respectively. Region II fields can be expressed as a linear combination of hybrid modes of a dielectric loaded waveguide of radius b [1], and core dielectric of radius a. The transverse fields in each of these regions are therefore expressible as:

$$\bar{E}_I = -\sum_i a_i \hat{e}_i^I \sin h\gamma_i^I z \qquad 0 < z < t \qquad (1-a)$$

$$\bar{H}_I = \sum_i a_i \hat{h}_i^I \cos h\gamma_i^I z \qquad 0 < z < t \qquad (1-b)$$

$$\bar{E}_{II} = \sum_i \hat{e}_i^{II} \left[b_i \cos h\gamma_i^{II}(z-t) - c_i \sin h\gamma_i^{II}(z-t) \right]$$

$$t < z < t + \ell \qquad (1-c)$$

$$\bar{H}_{II} = \sum_i \hat{h}_i^{II} \left(-b_i \sin h\gamma_i^{II}(z-t) + c_i \cos h\gamma_i^{II}(z-t) \right]$$

$$t < z < t + \ell \qquad (1-d)$$

$$\bar{E}_{III} = \sum_i d_i \ \hat{e}_i^{III} \sin h\gamma_i^{III}(L-z) \qquad t + \ell < z < L \qquad (1-e)$$

$$\bar{H}_{III} = -\sum d_i \hat{h}_i^{III} \cos h\gamma_i^{III}(L-z) \qquad t + \ell < z < L \qquad (1-f)$$

where $\left(\hat{e}_i^I, \hat{h}_i^I, \gamma_i^I\right)$, $\left(\hat{e}_i^{II}, \hat{h}_i^{II}, \gamma_i^{II}\right)$ and $\left(\hat{e}_i^{III}, \hat{h}_i^{III}, \gamma_i^{III}\right)$ are the transverse electric, transverse magnetic fields and the propagation factors of the ith mode in regions I, II and III respectively. Explicit Expressions for these quantities can be found in Reference [5].

The continuity of the transverse field components at the interfaces and $z = t$ and $z = \ell + t$ require:

$$\bar{E}_I = \bar{E}_{II}|_{at\ z=t} \qquad (2-a),$$

$$\bar{H}_I = \bar{H}_{II}|_{at\ z=t} \qquad (2-b)$$

and

$$\bar{E}_{II} = \bar{E}_{III}|_{at\ z=t+\ell} \qquad (2-c),$$

$$\bar{H}_{II}|_{at\ z=t+\ell} = \bar{H}_{III}|_{at\ z=t+\ell} \qquad (2-d)$$

Taking the cross products of: (2-a) with $\hat{h}_j^I, \hat{e}_j^I$ with (2-b), (2-c) with $\hat{h}_j^{III}, \hat{e}_j^{III}$ with (2-d); integrating each of the resulting equations over the cross section of the resonator and invoking the orthogonality properties of the eigenmodes, an infinite system of linear homogeneous equations for the unknowns a_i, b_i, c_i and d_i is obtained. The characteristic equation for the resonant frequencies is obtained by equating the determinant of a truncated subsystem of N equations from this system of equations to zero. Once the resonant frequencies are determined, they are substituted back in the homogeneous system, which is then numerically solved for all the coefficients (a_i, b_i, c_i, and d_i) in terms of one of them. Determination of the unloaded Q's involves the calculation of the energy (U) stored in the structure and the power (W_L) lost in the metallic walls, ground plane, the dielectric substrate and the dielectric resonator. The unloaded Q is the calculated from:

$$Q = \frac{\omega_0 U}{W_L} \qquad (3)$$

where ω_0 is the resonant frequency. Although the calculations of the Q is conceptually simple, the details are rather involved, principally because the non orthogonality of the hybrid eigenmodes. The method is summarized briefly.

First the energy stored in the structure (U) is computed as the sum of the stored energies in each of the regions I, II and III:

$$U = U_I + U_{II} + U_{III} \qquad (4)$$

where

$$U_i = \frac{\epsilon_{r_i}}{2} \int_{Vol_i} \bar{E}_i \cdot \bar{E}_i^* \qquad i = I, II, III \qquad (5)$$

$\bar{E}_i$ being the fields of eqn(1) in each of the 3 regions.

Second, the power lost in the enclosure walls, the ground plane, the dielectric substrate and the resonator are computed based on the low-loss assumptions that the fields are unperturbed by the losses. The surface current density in the enclosure walls and ground plane is $\hat{n} \times \bar{H}$, where $\hat{n}$ is the unit normal to the surface and $\bar{H}$ is the magnetic field. The power loss due to surface currents is then:

$$W_L = \frac{Rs}{2} \sum_i \int_S (\hat{n}_i \times \bar{H}_i) \cdot (\hat{n}_i \times \bar{H}_i^*) dS \qquad (6)$$

The dielectric loss in the substrate and resonator is simply obtained as the product of the corresponding loss tangent, times the electric energy stored in the medium.

3. Results

Fig. 2 shows the unloaded Q variation with the substrate thickness of a millimeter wave dielectric resonator. The substrate material is taken to be GaAs ($\epsilon_r = 12.9$, $\tan\delta$=0.001 at 33 GHz) and the resonator is a high Q material ($\epsilon = 24$, $\tan\delta = 0.0002$ at 33 GHz). The resonator diameter was kept fixed at .1″, and its height was varied to maintain the frequency constant at 33 GHz, when the substrate thickness was varied. The ground plane and enclosure are assumed to be gold plated ($\sigma = 4.1\times10^7$ mhos /cm). Four modes were considered: $TE_{01\delta}$, $TM_{01\delta}$, and the hybrid HE_{11} , and HE_{12} modes. For each mode the contributions to the loss of the ground plane, the substrate, the dielectric resonator, the sides of the metallic enclosure and the cover were computed separately. Generally, the most significant contribution to the losses are the ground plane and the dielectric resonator. However, as the substrate's thickness is increased, its loss contribution becomes very significant. The unloaded Q of the hybrid modes are typically 30 % to 60 % higher than the $TE_{01\delta}$ mode. Figures 3 and 4 show the same data for an Alumina and Quartz substrates respectively.

Computed and measured results of the unloaded Q's of the four lowest order modes of a C-band resonator are compared in Table 1. The calculations and measurements of the unloaded Q's show excellent agreement. Fig. 5 shows the unloaded Q's variation with the length of the enclosure for different modes, for the same dielectric resonator. As the length of the enclosures increases the unloaded Q's increases as expected.

4. Conclusions

A method for calculating the unloaded Q of hybrid modes in dielectric loaded resonator has been described. The method was applied to a 33 GHz resonator on a GaAs, Alumina and Quartz substrates. The contribution of the losses in each of the resonator's elements are computed separately. It is shown that the hybrid modes can have substantially higher Q's than the TE or TM modes (except for the Quartz substrates). This result is particularly significant for millimeter waves.

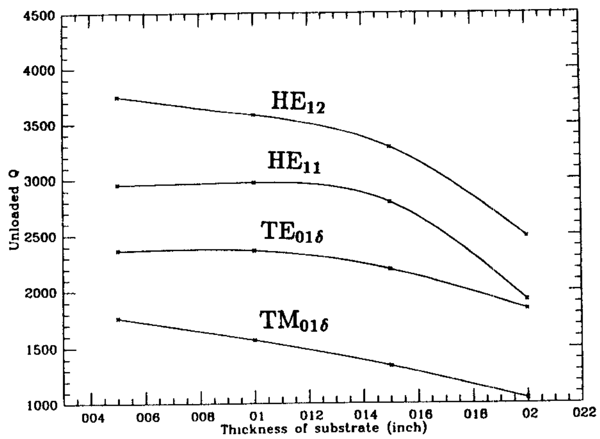

Fig. 2 Unloaded Q's vs. substrate thickness of the lowest four modes of a 33 GHz dielectric resonator on GaAs substrate.

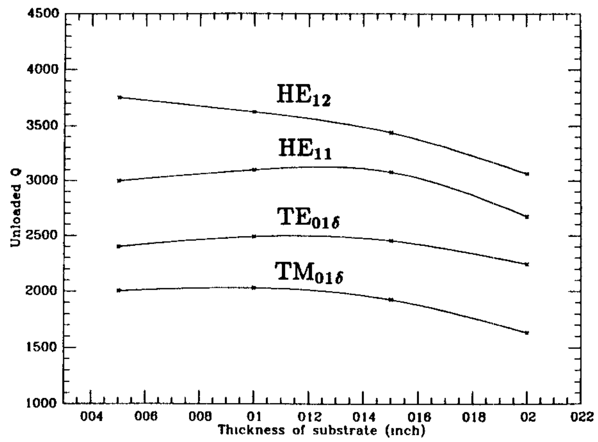

Fig. 3 Unloaded Q's vs. substrate thickness of the lowest four modes of a 33 GHz dielectric resonator on Alumina substrate.

Table 1. Computed and Measured Results For A C-Band Resonator

$a = .34'', b = .51'', \epsilon_{r_1} = 35.74, \tan\delta = .00012, \epsilon_{r_2} = \epsilon_{r_3} = 1.$
$t = .15'', h = .3'', L = .6''$

Mode	Resonant Frequency (GHz) Computed	Resonant Frequency (GHz) Measured	Unloaded Q Computed	Unloaded Q Measured	%Error
TE_{01}	3.4305	3.4374	6160	6138	0.36%
TM_{01}	4.5461	4.5329	9050	8472	6.82%
HE_{11}	4.2260	4.2243	7650	7750	-1.29%
HE_{12}	4.3162	4.3285	7560	7274	3.93%

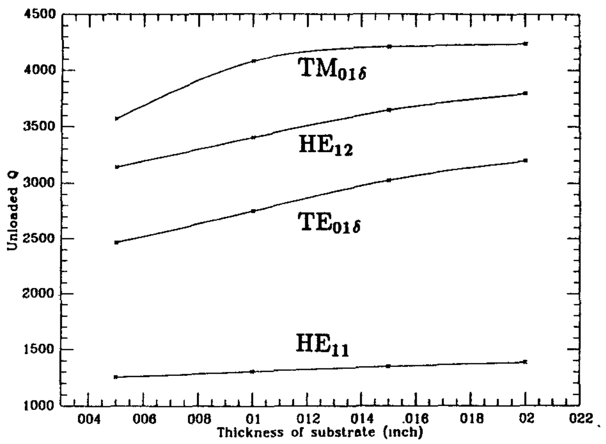

Fig. 4 Unloaded Q's vs. substrate thickness of the lowest four modes of a 33 GHz dielectric resonator on Quartz substrate.

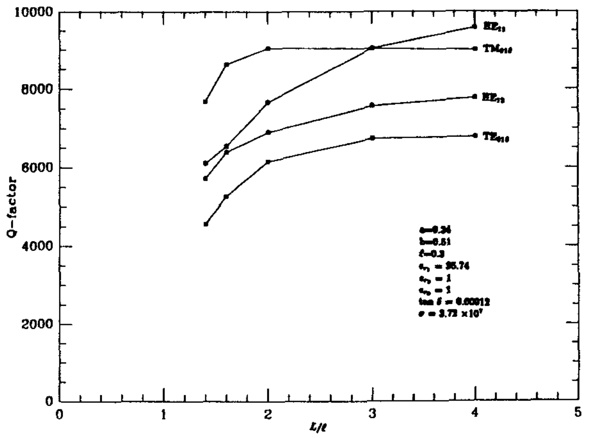

Fig. 5 Unloaded Q's variations with the length of the enclosure for the four lowest order modes of a C-band resonator.

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